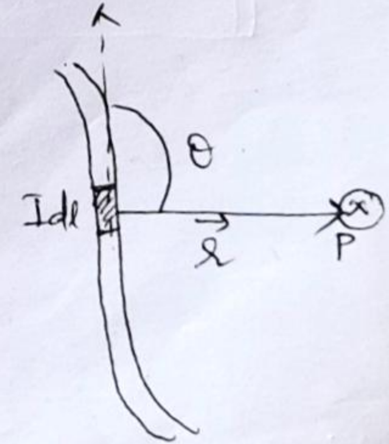
BIOT - SAVART's LAW - mag. field due to a current element

Consider an ~~infinitesimal~~ infinite conductor XY carrying current I. Consider an infinitesimal element dl. The magnetic field $d\vec{B}$ due to this current element at a point P can be determined by Biot savart's Law.



Biot Savart's Law:

According to Biot-Savart's law, the magnitude, magnetic field dB is proportional to the current, the element dl and inversely proportional to the square of the distance 'r' Its direction is perpendicular to the plane containing $\vec{dl}$ and $\vec{r}$.

$$\vec{dB} \propto \frac{I\vec{dl} \times \vec{r}}{r^3}$$

$$= \frac{\mu_0}{4\pi} \frac{I\vec{dl} \times \vec{r}}{r^3} = \frac{\mu_0}{4\pi} \frac{I\,dl \sin\theta}{r^2}.$$

$\mu_0/4\pi \rightarrow$ Constant of proportionality.

$$\frac{\mu_0}{4\pi} = 10^{-7}\ \mathrm{TmA}$$

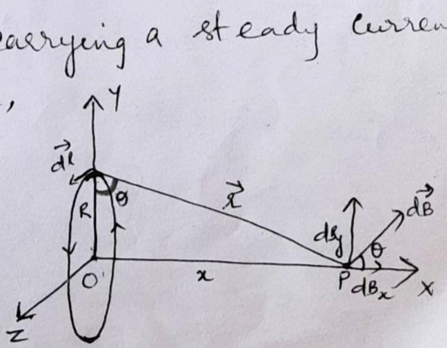
$\mu_0 \rightarrow$ permeability of free space. ($4\pi \times 10^{-7}$ TmA)

* There is an angle dependence in Biot Savart's law. The mag. field along the direction of the wire is zero. [$\because \theta = 0$, $\sin\theta = 0$]

* Field is of long range, obeys inverse square law. The principle of superposition is applicable

Magnetic field along the axis of a circular current loop

Consider a circular loop carrying a steady current] The loop is placed in the Y-Z plane, with its centre O at the origin of radius R. The axis of the loop is along the X-axis. Consider any arbitary P along the axis at a distance 'x'



from its centre. $\vec{dl}$ is a small line element of the loop. The magnitude dB of the magnetic field due to dl is given by Biot-Savart law.

$$dB = \frac{\mu_0}{4\pi} \cdot \frac{I dl \sin\theta}{r^2}$$

As the element dl is in the y-z plane where as $\hat{r}$ (displacement vector $\vec{r}$) is in the x-y plane, $\therefore \sin\theta = 1$, as $\theta = 90°$.

$$\therefore dB = \frac{\mu_0}{4\pi} \cdot \frac{I dl}{r^2} = \frac{\mu_0}{4\pi} \frac{I dl}{(x^2+R^2)} \qquad \because r^2 = x^2 + R^2$$

The direction of $\vec{dB}$ is shown in fig which can be resolved into parallel and perpendicular components dB_x and dB_y. When all the perpendicular components 'dB_y' are summed over, they cancel out because of the contribution of diametrically opposite dl element. Hence the net field is that due to parallel components along x-axis.

$$\therefore dB_x = dB \cos\theta$$

from fig: $$\cos\theta = \frac{R}{(x^2+R^2)^{1/2}}$$

$$\therefore dB_x = \frac{\mu_0}{4\pi} \cdot \frac{I dl}{(x^2+R^2)} \cdot \frac{R}{(x^2+R^2)^{1/2}}$$

$$= \frac{\mu_0}{4\pi} \cdot \frac{I dl\, R}{(x^2+R^2)^{3/2}}$$

Summing over all the $dl \rightarrow$ circumference of loop

$$\therefore B = \frac{\mu_0 I R}{2 \cancel{4\pi} (x^2+R^2)^{3/2}} \cdot \cancel{2\pi} R$$

$$B = \frac{\mu_0 I R^2}{2(x^2+R^2)^{3/2}} \hat{i}$$ (along the x-axis)

At the centre of the loop, $x = 0$.

Then

$$B = \frac{\mu_0 I \cancel{R^2}}{2 R^{\cancel{3}}} = \frac{\mu_0 I}{2R} \hat{i}$$

The direction of magnetic field is given by right hand thumb rule.

[If the wire is of semi circle shape, then at the centre

$$B = \frac{\mu_0 I}{4R}$$]

Ampere circuital law states that the line integral of magnetic field $\vec{B}$ taken over a closed loop is equal to 'μ_0' times the total current I threading the loop.

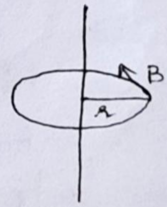
Mathematically $\oint \vec{B} \cdot \vec{dl} = \mu_0 I$

* The above relation is independent of shape or size of the closed loop enclosing the current
* The direction of current / magnetic field can be deduced by Right hand thumb rule.
* At each point on the loop chose (amperian loop). $\vec{B}$ is tangential to the loop. $\vec{B}$ is zero whenever $\vec{B}$ is considered normal to the loop.
* Ampere circuital law can be applied conveniently to any system with a symmetry.
* If the length of the amperian loop for which the magnetic field is tangential is 'L', then

$$BL = \mu_0 I$$

or

$$B = \frac{\mu_0 I}{L}$$



* For a circular amperian loop, which is at a distance 'r' outside the current carrying wire then

$$B(2\pi r) = \mu_0 I$$

$$B = \frac{\mu_0 I}{2\pi r}$$

which is equivalent to the expression of magnetic field due to a straight current carrying wire at a distance 'r' from its centre.

* It implies that the field at every point on a circle of radius r, has same magnitude i.e. it posses cylindrical symmetry.

<u>SOLENOID</u> : A solenoid is a device which generates magnetic field when current is passed through it. It is a kind of cylindrical coil having tightly wound insulated/enamalled wires in the form of helix.

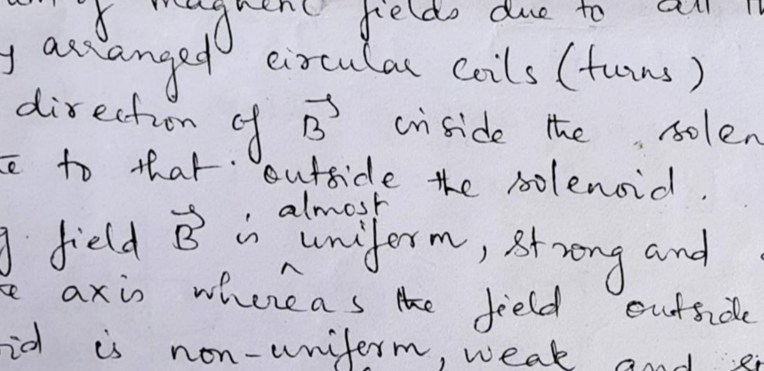
→ The neighbouring turns are closely spaced so that each turn can be considered as a circular loop

→ Used in TV, fan etc.

→ The magnetic field around a solenoid, is the vector sum of magnetic fields due to all the linearly arranged circular coils (turns)

→ The direction of $\vec{B}$ inside the solenoid is opposite to that outside the solenoid.

→ mag. field $\vec{B}$ is almost uniform, strong and directed along the axis whereas the field outside the solenoid is non-uniform, weak and spreads around the solenoid. (The mag. field outside an ~~solen~~ ideal solenoid is practically zero).



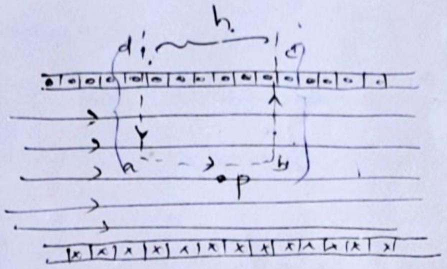
magnetic field produced by a current

Magnetic field at a point ~~bte~~ inside the solenoid.

* Ampere circuital can be applied as the system posses symmetry.

Consider a long straight solenoid of length 'L', 'N' no. of turns which are tightly packed. Due to a current I, a strong uniform magnetic field $\vec{B}$ inside the solenoid as shown in fig.

• Q



$BL = \mu_0 N I$

Consider ~~an~~ rectangular amperian loop abcd along cd (outside), the field is almost zero. [the field is along the axis with no perpendicular component and the field outside it approaches zero]. Sections bc and ad does not contribute to the magnetic field as they ~~are~~ perpendicular components ~~to the~~ of B is zero. ∴ the field exists only along the section ab of length 'h'. Let 'n' be the no. of turns per unit length. Then total no. of turns enclosed by length h is nh and the enclosed current is $I_e = nh$.

By Ampere circuital law,

$n \rightarrow$ no. of turns per unit length of the solenoid

$$\therefore \quad BL = \mu_0 I_e$$

$$\text{or} \quad Bh = \mu_0 (nh) I$$

$$\boxed{B = \mu_0 n I}$$

* The direction of the field is given by right hand thumb rule.

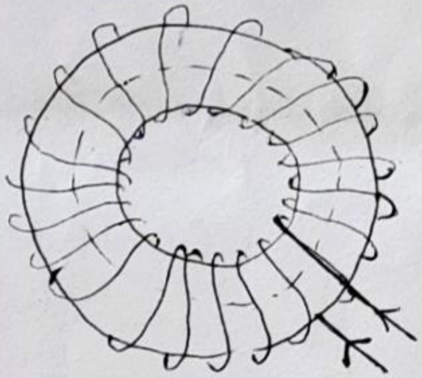
* The above eqn holds good for points well inside the solenoid, but at a point near the ends of a solenoid, the magnetic field is nearly equal to $\frac{1}{2}(\mu_0 n I)$

TOROID

A toroid is a special kind of solenoid which has been bent into a circular shape to close itself. It also generates a uniform, strong magnetic field when current is passed through it.

* Toroid is a hollow circular ring on which a large no. of turns of a wire are closely wound as shown in fig (a)

* fig (b) gives the sectional view



(a)

$2\pi r_2$

$I_e = I$ current in the toroid,

$r_2 = r$ avg. radius of the toroid.

$$\therefore \boxed{B = \frac{\mu_0 N I}{2\pi r} = \mu_0 n I} \quad (1)$$

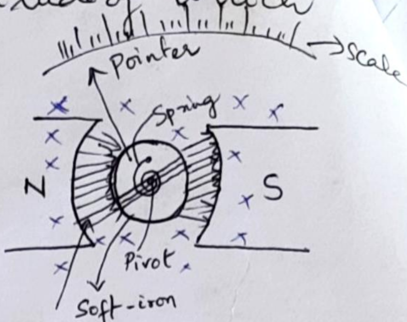
where $n = \frac{N}{2\pi r}$ no. of turns/unit length of the toroid

Eqn (1) shows that the mag. field is equivalent, to that produced by a solenoid

MOVING COIL GALVANOMETER

: It is a device to detect current in a circuit.

Principle: A current carrying coil placed in a mag. field experiences a torque, the magnitude of which depends on the strength of current.



schematic diagram of a moving coil galvanometer

Construction: A weston (pivoted type) galvanometer consists of a rectangular coil of insulated copper wire wound over an aluminium frame. The motion of the coil is controlled by a pair of hair springs of phosphor-bronze. The spring provides the restoring torque and serve as current leads. An aluminium pointer is attached to the coil which shows the deflection on a scale. The coil is placed between concave poles of a permanent magnet. There is a soft iron core (cylindrical) which makes the field radial and also increases the strength of the mag. field.

Theory and working

As the field is radial, the ~~plane~~ area vector $\vec{A}$ of the coil remains perpendicular to the mag. field $\vec{B}$. When a current I flows through the coil, a torque acts on it deflecting it.

$$\tau = NIAB \sin 90^\circ = NIAB$$

(N - no. of turns of the coil
A - area of the coil)

The torque 'τ' deflects the coil through an angle ϕ. A restoring torque is set up in the coil due to elasticity of the spring given by

$\tau = k\phi$ where 'k' is the torsional const or restoring couple per unit twist.

At equilibrium,

Restoring torque = Deflecting torque

$$k\phi = NIAB$$

$$\phi = \left(\frac{NAB}{k}\right) I \qquad \therefore \phi \propto I$$

Thus deflection produced in the galvanometer is proportional to the current flowing through it.

Figure of merit of a galvanometer :
It is defined as the current which produces a unit scale deflection in the galvanometer.

$$k' = \frac{I}{\phi} = \frac{k}{NAB}.$$

current sensitivity : It is defineds as the deflection produced in the galvanometer when a unit current flows through it. Current sensitivity, $I_s = \frac{\phi}{I} = \frac{NAB}{k}$

Also $I_s = \frac{1}{\text{fig. of merit}}$

Voltage sensitivity ! It is defined as the deflection produced in the galvanometer when a unit potential difference is applied across its ends.

$$\text{Voltage sensitivity}, V_s = \frac{\phi}{V} = \frac{\phi}{IR} = \frac{NBA}{kR} = \frac{I_s}{R}$$

ie $V_s = \frac{I_s}{R}$

* Increasing the current sensitivity may not increase the voltage sensitivity. Suppose we double the no. of turns to double current sensitivity ie

$$N \rightarrow 2N,$$

so that $\frac{\phi}{I} \rightarrow 2\frac{\phi}{I}$

But the resistance of the galvanometer is likely to double as it depends the length of the wire

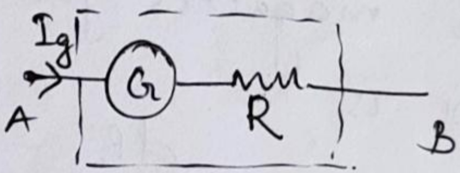
$\therefore$ $N \rightarrow 2N$ then $R \rightarrow 2R$

Thus, $\frac{\phi}{V} \rightarrow \frac{\phi}{V}$ remains unchanged.

Conversion of Galvanometer to voltmeter (contd).

A galvanometer can be converted to a voltmeter by connecting a large resistance in series with it because an ~~galvano~~ ideal voltmeter high or infinite resistance so that it measure the potential difference across its ends correctly.

Let G = Resistance of the galvanometer



I_g = current with which galvanometer gives full deflection

$I_g = nk$ (k is the fig. of merit).

$0 - V$ = required range of the voltmeter

R = high resistance

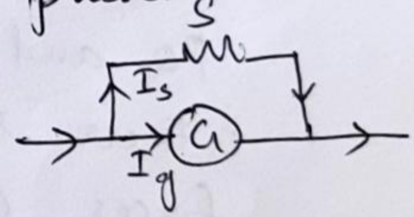
Total resistance $= R + G$

$$I_g = \frac{V}{R+G}$$

$$\therefore R = \frac{V}{I_g} - G$$

Conversion of Galvanometer to Ammeter

* A Galvanometer is a sensitive device which detects current of the order of 'μA' and it has appreciable resistance 'G'.
* An ideal Ammeter has zero or negligible resistance such that it measure the current flowing through it correctly.
* A (G) can be converted to an (A) by connecting a very low resistance or shunt resistance parallel to it such that net resistance of the combination is reduced and most of the current passes thru' it.



Here $I_s = I - I_g$.

Value of shunt, $S = \frac{I_g}{I - I_g} \times G$

$G \rightarrow$ galvanometer resistance

$I_g \rightarrow$ current that gives full scale deflection